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Jeremy Gray. *Henri Poincaré: A Scientific Biography*. Princeton and Oxford: Princeton University Press, 2013. xiv + 594 pp. Figures, notes, appendix, bibliography, and index. $35.00 U.S. (cl). ISBN 9-78-069115271-4; $35.00 U.S. (eb). ISBN: 9-78-140084479-1.

Review by Daniel S. Alexander, Drake University.

Near the beginning of his *Everything and More*, David Foster Wallace anoints Georg Cantor the best mathematician of the nineteenth century.[1] Wallace's claim is no doubt intentionally provocative, and in that spirit I am going to suggest he had it wrong: the French mathematician Henri Poincaré would have been a far better choice. Poincaré profoundly influenced developments in a myriad of fields including geometry, topology, analysis, algebra, philosophy, and in several areas of mathematical physics. As if that were not enough, he also had a hand in the outcome of the Dreyfus Affair.

It is an impressive resume, one that led E. T. Bell, whose writings about mathematicians tend towards the hagiographic, to call him the "The Last Universalist." While that title may be a bit hyperbolic, the suggestion that he had a command of the field of mathematics and physics that is nowadays most rare is no exaggeration. Poincaré was born in Nancy in 1854 and died unexpectedly at home in Paris in 1912 from an embolism during his recuperation from prostate surgery. At the time of his death, he was a public figure in France, due to his many popular essays about science and mathematics, as well as to his scientific accomplishments. Nowadays, he is probably best known among the general public for an eponymous conjecture that states that any bounded three-dimensional body without any holes can be continuously transformed into a sphere but not into a body with any holes, such as a torus or a pretzel.

A consequence of this conjecture would be that you could not mash a donut into a ball without tearing the dough at some point since that tear would represent a discontinuous action.[2] In other words, if you have an infinitely malleable rabbit (and ignore its holes) you can mathematically transform it into a sphere without ripping it apart. If true, this conjecture would allow mathematicians to definitively classify three-dimensional objects by the number of their holes. Although the conjecture is simple to describe, it is devilishly difficult to prove by the rigorous standards of mathematics (a rigor to which Poincaré did not always fully adhere). And therein lay the conjecture's lure. The conjecture attached to Poincaré's name was finally proved to be true in the early 2000s--about 100 years after Poincaré allegedly made it--by the Russian mathematician, Grigori Perelman, who gained notoriety (and a featured role in a 2006 New *Yorker* article [3]) for refusing the prestigious Fields Medal which was awarded to him for his accomplishment.[4] Four years later Perelman doubled-down when he refused the Millennium Prize--and the $1,000,000 that came with it--that the Clay Institute had offered to anyone solving the Poincaré conjecture.[5]

To adequately explain the context of the Poincaré conjecture within Poincaré's corpus to someone not schooled in mathematics is a difficult proposition. This is compounded by the fact that the body of mathematics that Poincaré created is not only beyond what a reasonably mathematically literate person, say someone who has taken a couple of years of calculus and perhaps a differential equations course, would have been exposed to, but would also prove challenging to many mathematicians. This makes the task of anyone attempting a fully-blown scientific biography of Poincaré--which is what Jeremy Gray, one of the pre-eminent current practitioners of mathematical history not only undertakes but also succeeds at--quite formidable.

I do not mean to suggest that Gray's book is about the Poincaré conjecture. Indeed, it is a testament to the breadth of Poincaré's knowledge that Gray only devotes a tiny fraction of his book's approximately 550 pages to it. Rather, he provides a rich, mathematically detailed account of the entirety of Poincaré's scientific life. Since a good portion of the book is quite technical and perhaps best understood by those already knowledgeable about the fields to which Poincaré contributed, the primary audiences of Gray's book are mathematical historians and mathematicians who want to know more about Poincaré. That said, the structure of Gray's book provides a clear path that should interest a reader curious about the mathematical, scientific and philosophical currents of the dawn at the twentieth century, even one who has not taken a calculus class. This path, which takes up about one-half of the text, consists of the introduction, the book's first two chapters, as well as its final two, and expertly (and accessibly) places Poincaré within the scientific culture of the late nineteenth and early twentieth centuries. As a bonus, the attentive reader will also encounter illuminating discussions about specific areas of mathematics, particularly non-Euclidean geometry whose development is perhaps best viewed as a manifestation of modernism (the subject of Gray's *Plato's Ghost*).[6]

The book's middle chapters compose the technical path, and in it, Gray goes into much more detail about the mathematics involved. Readers without a good deal of mathematical knowledge may find the going rough. However, there is still much a general reader can gain from it. For example, below I refer to a major prize competition, the King Oscar Prize, that Poincaré won. While Gray analyzes the mathematics of Poincaré's entry in fine detail, he weaves in a rich discussion of the politics of the prize. A mathematical non-expert who is nonetheless interested in the economy of a scientific prize should not hesitate to skip the math and focus on the human interactions.

Poincaré's work grew out of his conception of the space we inhabit, which he argued was necessarily subjective and ultimately unknowable. While he argued that we could never truly understand the nature of space, our understanding of it is best discerned from the movement of objects through it. As such, whether one sees space as Euclidean or not is in many senses a matter of convention or convenience. Gray examines and elucidates Poincaré's ideas about space and, in terms a general audience can understand, shows how it informed his mathematics, philosophy and physics. Although Poincaré participated in the emerging discussions regarding logic and its role in mathematics (he conversed with Bertrand Russell and others), he valued clarity above all and often relied on mathematical arguments that sacrificed a little rigor for clarity.

Poincaré embraced many new ideas in mathematics--especially those of Georg Cantor and Sophus Lie--yet avoided others such as Henri Lebesgues's revolutionary theory of measure. Gray also suggests that he had a distaste for some of the fascinating new mathematical objects that were beginning to appear, which he considered somewhat artificial since they seemed to him to spring from the minds of mathematicians trying to create bizarre objects, as opposed to arising organically from a mathematical theory or an experience with objects in space. This reaction is somewhat ironic since it turns out that some of these "pathological" examples occur quite naturally in the theory of chaos, some of whose core ideas Poincaré was among the first to explore. Despite following his own nose, and largely ignoring mathematicians whose work did not suit him, Poincaré by no means worked in isolation. He developed strong connections with many mathematicians and, given his own strong opinions and unique way of doing things, came into conflict with others. Gray describes and analyzes these networks of mathematicians and scientists that formed around (and sometimes in opposition to) Poincaré, and in so doing, provides a snapshot of the field of mathematics in the late nineteenth and early twentieth centuries, principally in Europe, but also in the United States.

Poincaré's methodology sometimes led to mistakes (which are more common in mathematics than the non-specialist might suspect), and Gray does not shy away from discussing them, noting at one point that it would be rare for one of Poincaré's calculations to be error-free. One of Poincaré's mistakes occurred in his first-place entry in a mathematical contest sponsored by King Oscar II of Sweden in the 1880s. The prize announcement in 1885 specified that entrants tackle one of four problems, any one of which, Gray observes, served as an invitation for Poincaré to enter. The one Poincaré chose concerned a topic no less fundamental than the stability of the solar system. While it did not affect the awarding of the prize or the scope of his work, Poincaré's mistake was serious, and he was unable to completely rectify it. Gray, who has written previously on the role of mathematical contests,[7] offers a fascinating study of the dealings surrounding the competition. For example, the choice of the judges turned on the fact that mutual antipathies were so strong that certain pairs of mathematicians could not be counted on to cooperate on judging the entries, much less agree on a victor. He also investigates how those involved in the prize dealt with Poincaré's mistake, which had the potential to be embarrassing.

The view of Poincaré that emerges from Gray's book is that of a supremely gifted creator, one whose work is *sui generis*. Gray's words may give the reader a stronger sense of this than my own: speaking of his first major work, the series of papers in the early-to-mid 1880s mentioned above, Gray observes,

"The idea of automorphic functions was essentially new--no one had seen this generalization of the elliptic and modular functions before or even sought one--and so was the geometry of Jordan curves. No one had thought to exploit non-Euclidean geometry in this fashion. No one had treated differential equations in this broad setting before. Any one of these achievements, had they been isolated, would have impressed Poincaré's peers. Taken together, they made his name among mathematicians" (p. 246).

Gaston Darboux, one of the most prominent--both creatively and institutionally--French mathematicians of the time made a similar observation about Poincaré's doctoral thesis, noting that it contained enough good ideas for several dissertations. Poincaré's productivity continued unabated over the next three decades with fundamental studies in several areas of mathematics including the wonderfully named "celestial mechanics" (the study of the motions of the heavens) and topology (the study of, among other things, intrinsic properties of a mathematical object that do not change under a continuous transformation--it is in this area that the Poincaré conjecture arises). These later works continue to be characterized by a fashioning of existing ideas in a completely novel way, and by the invention of new mathematics. His pursuit of mathematics led him to mathematical physics where he investigated electromagnetism, the composition of matter, quantum theory, relativity, and influenced students throughout the world via a series of textbooks stemming from his lectures. A contingent of French mathematicians and physicists spearheaded his nomination for the 1910 Nobel Prize for physics. While it was a reasonable nomination, it was contested, and he was not awarded a Nobel. Gray provides both a political and scientific analysis of why he did not win.

Poincaré was active in both logic and the philosophy of science and mathematics, and Gray's analysis of Poincaré's contributions provides a nice overview of the philosophical discussion of the late nineteenth and early twentieth centuries involving mathematics. Poincaré was particularly involved in the debate regarding the logical foundations of mathematics, which led him to engage Bertrand Russell and Louis Couturat in the early 1900s, and later, David Hilbert and Ernst Zermelo. While Poincaré found much to value in their works, he evidently did not think they were successful in their attempts to deduce mathematics from logic. Quoting Gray:
"In this spirit, Poincaré bridled at the idea that such postulates can bring mathematical objects into existence. To him, it seemed that logic had become creative by giving out names" (p. 143).

I have suggested several times that a reader interested in the practice of mathematics might find Gray's book compelling reading. That begs a question: why would one want to know more about mathematics? One reason might lie in the fact that mathematics is an art of creation, which, much like poetry or painting, seeks to give an account of the world as the practitioner sees it. Obviously, most people find painting and poetry more accessible than mathematics, which makes it more difficult for a general reader to see mathematics from this perspective. This difficulty is perhaps exacerbated because the mathematician as aesthetician is not present in most people's encounters with mathematics. Given the emphasis on accountability and assessment in education, the concept of mathematics as an art often gets lost in the rush to provide students with practical, measurable skills.

Gray's presentation of Poincaré is, at its core, aesthetic. One gets a sense of why Poincaré's mathematics evolved as it did, much as good biographies or exhibitions of painters help one to see clear lines of evolutions in their work. Moreover, as with many artists, it is hard to imagine Poincaré doing anything else but pursing mathematics. A good intellectual biography of an artist should help the reader see how a particular worldview shapes the pursuit of art. Gray's book does that most admirably.

NOTES

[1] David Foster Wallace, *Everything and More: A Compact History of Infinity* (New York: W.W. Norton, 2003).

[2] Some readers may remember a faux counter-example to the Poincaré conjecture that Stephen Colbert gave on television, where he purported to transform a torus into a sphere by mashing a donut into a ball.
(see: \\http://www.colbertnation.com/the-colbert-report-videos/73352/august-22-2006/cheating-death---fields-medal, 2013).

[3] Sylvia Nasar and David Gruber, "Manifold Destiny," *The New Yorker* (August 28, 2006): 44-57.

[4] As he noted in the video referenced in note one, Colbert was only too willing to accept his prizes in his stead.

[5] Gray points out that the term "Poincaré conjecture" is actually a misnomer since it was more of a question that Poincaré posed rather than something he conjectured to be true.

[6] Jeremy Gray, *Plato's Ghost: The Modernist Transformation of Mathematics* (Princeton, N.J., and Oxford: Princeton University Press, 2008).

[7] Jeremy Gray, ``A History of Prizes in Mathematics," in J. Carlson, A. Jaffe and A. Wiles, eds., *The Millennium Prize Problems* (Cambridge, Mass. and Providence, R.I.: Clay Mathematics Institute and American Mathematical Society, 2006), pp. 3-27.

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